4.2 Completed Notes

4.2: Prime and Composite Numbers

Definition: A prime number is a number with exactly two distinct positive factors, namely 1 and themselves.

Definition: A composite number is a number with more than two distinct positive factors:

Is 1 a prime number or a composite number?

Neither, it has only 1 factor.

Find which numbers are prime in the set {1, 2, ..., 100}

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

This is known as the Sieve of Eratosthenes.

Theorem: If n is composite, then it has a prime factor p with the property that $p^2 \leq n$.

In other words, to see if a number is prime, we need only check all of the possible prime factors up to its square root.

Proof: Let p be the smallest divisor of n that is not 1. Then there is a number k such that $pk = n$. $(k = \frac{n}{p})$. $k \geq p$ since p is the smallest divisor. So,

$$n = pk \geq p \cdot p = p^2, \text{ so } p^2 \leq n. \blacksquare$$

Example: List the factors of 28. Is 28 prime or composite?

1, 2, 4, 7, 14, 28 composite

Is 301 prime? $\sqrt{301} \approx 17.35$

~~2~~ 7 17

~~3~~ 11

~~5~~ 13 $7 \mid 301$, so 301 is composite

Is 307 prime? $\sqrt{307} \approx 17.52$

~~2~~ ~~7~~ ~~17~~

~~3~~ ~~11~~

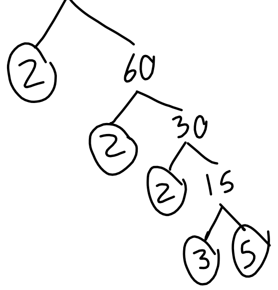
~~5~~ ~~13~~ 307 is prime

Fundamental Theorem of Arithmetic: Each composite number can be written as a product of primes in exactly one way (ignoring the order of the factors).

Definition: This product described above is known as the prime factorization of a number.

Example: What is the prime factorization of 120?

$120 = 2^3 \cdot 3 \cdot 5$



Example: What is the prime factorization of 270?

$270 = \boxed{2 \cdot 3^3 \cdot 5}$